Bipartite Correlations through Enriched System Information in Phase-Coherent Amplitudes.

Jorgen Karlsen

Abstract: This work introduces a refined kinematic framework that models the physical state of an individual particle as a continuous, phase-coherent directional amplitude, defined by a signed distribution of amplitude vectors on the unit sphere. This approach expands the standard theoretical toolkit by enabling the precise tracking of individual particle properties—specifically the internal geometric phase and directional orientation—through independent projection operations. By retaining this enriched phase information prior to probability assignment, the framework natively resolves interference effects that are typically lost in standard discrete algebraic factorizations. To rigorously benchmark this phase-resolved model, we apply it to bipartite correlations in photon pairs. We demonstrate that modeling measurements as geometric overlaps of individual subsystem amplitudes naturally reproduces the full quantum correlation statistics, exactly saturating the Tsirelson bound ($2\sqrt{2}$). These results show that bipartite quantum correlations can be represented explicitly in terms of phase-coherent continuous amplitude distributions offering more information about the process of reproducing the Tsirelson bound.

1. Introduction

The quantum mechanical description of spin is traditionally formulated using discrete algebraic labels and operator algebra applied to ensembles. While this formalism is empirically successful, it effectively treats the state of an individual particle between preparation and measurement as a "black box," remaining largely silent about its internal geometric structure. In this work, we investigate whether this standard algebraic description represents a kinematic coarse-graining that overlooks information relevant for calculating specific correlations.

We explore the possibility that the spin state of an individual particle can be described more completely by a phase-coherent directional amplitude. In this refined picture, each particle carries a definite directional amplitude with an associated internal phase, consistent with the rotational symmetry of SU(2). To ground this definition physically, we re-analyze the Stern–Gerlach experiment [1], identifying the specific post-selection distribution required to reproduce the geometric transition probabilities discussed by Feynman [2]. We find that an expanded definition of spin—one that treats amplitude density as a signed, phase-coherent distribution—is necessary to capture the full rotational structure of the single particle.

In this framework, measurement outcomes are not merely selected from a pre-existing set of eigenvalues but are generated by projections of this internal amplitude structure onto the analyzer settings. Statistical predictions then emerge from distributions over these phase-resolved amplitudes.

To rigorously validate whether this enriched kinematic description provides a faithful representation of quantum phenomenology, we apply it to the benchmark problem of entangled photon pairs. We ask a specific structural question: Can a strictly geometric framework, based on these enriched spin variables, reproduce the theoretical maximum for quantum correlations?

We show that this is indeed the case. For entangled photon pairs prepared with a shared geometric phase (orientation) at emission, we model the measurement process as an analyzer projection of the particle's internal amplitude. Correlations arise when the bilinear product of the responses is averaged over the shared preparation distribution. The resulting correlation function recovers the standard quantum cosine form and exactly saturates the Tsirelson bound [3].

This result suggests that the limitations typically associated with classical probabilistic models stem from the assumption that probabilities must factorize directly. By retaining phase coherence at the amplitude level prior to probability assignment, we recover the full quantum statistics within a geometrically consistent framework. This phase-resolved description offers a natural platform for understanding filtering, polarization, and interference within a unified kinematic setting.

2. Stern–Gerlach Revisited: Angular Amplitude Structure and Post-Selection

The Stern–Gerlach experiment [1] is commonly presented as a measurement revealing a pre-existing binary spin property. Operationally, however, the device functions as a state-preparation filter rather than a revelation of intrinsic values, as emphasized in both early and modern analyses: an initially unpolarized ensemble (beam) is spatially separated, and post-selection of only one output channel prepares a new spin state of the particles in this ensemble. Closely related geometric interpretations of spin preparation and rotation can be found in the analysis of spin and rotations presented in The Feynman Lectures [2], where the Stern–Gerlach experiment with its binary outcomes for spin-½ particles is treated as a paradigmatic example of quantum state preparation rather than classical orientation. In this section we show that this preparation can be represented by a **directional amplitude structure on the unit sphere**, whose properties reproduce the standard quantum transition probabilities under rotation, while retaining more specific information.

2.1 Isotropic ensemble and analyzer geometry

Consider an initially isotropic ensemble of spin-½ particles incident on a Stern–Gerlach analyzer oriented along the z-axis. Let θ denote the polar angle between an arbitrary unit vector $\hat{v}$ on the Bloch sphere and the analyzer axis $\hat{z}$.

Post-selection of the "spin-up" output defines a prepared state. We associate to this state a **directional amplitude density** distribution of individual particles on the unit sphere S^2,

$$\rho(\theta) = \frac{1}{4\pi} + \frac{1}{2\pi}\cos\,\theta. \tag{2.1}$$

This object satisfies normalization requirement,

$$\int_{S^2} \rho(\theta)\, d\Omega = 1, \tag{2.2}$$

but it is not everywhere non-negative. In particular, $\rho(\theta) < 0$ for $\cos\,\theta < -1/2$. This feature is essential and will be addressed below. As with other quasi-distributions in quantum theory, this object acquires operational meaning only under integration against measurement-defined regions.

2.2 Interpretation of the density

The function $\rho(\theta)$ is **not** interpreted as a classical probability density over hidden spin directions. Instead:

1. ρ is a **phase-weighted directional amplitude density**, encoding the biased preparation imposed by the analyzer.
2. Negative values are admissible and reflect **destructive interference** of amplitudes associated with different orientations.
3. Observable probabilities arise only **after integration over analyzer-defined regions**, not by pointwise evaluation of ρ.

Accordingly, ρ plays a role analogous to a quasi-distribution: it is a bookkeeping / tracking device for amplitudes prior to projection.

2.3 Second analyzer and hemisphere integration

Let a second Stern–Gerlach analyzer be oriented along a unit vector $\hat{n}$ making an angle α with the z-axis. The analyzer accepts those components whose projection onto $\hat{n}$ is positive. Geometrically, this corresponds to the hemisphere

$$S_{\hat{n}} = \{\hat{v} \in S^2 : \hat{v} \cdot \hat{n} \geq 0\}. \tag{2.3}$$

The transition probability for a particle prepared in the z-up state to be transmitted by the second analyzer is given by the integral of the amplitude density over this hemisphere:

$$P(\hat{z}\uparrow \rightarrow \hat{n}\uparrow) = \int_{S_{\hat{n}}} \rho(\theta)\, d\Omega. \tag{2.4}$$

2.4 Explicit evaluation

Choose coordinates such that $\hat{n}$ lies in the xz-plane at polar angle α. Writing $\hat{v} \cdot \hat{n} = \cos\ \gamma$, the hemisphere condition is $\gamma \leq \pi/2$.

The integral (2.4) separates into two terms:

$$\int_{S_{\hat{n}}} \rho\, d\Omega = \frac{1}{4\pi}\int_{S_{\hat{n}}} d\Omega + \frac{1}{2\pi}\int_{S_{\hat{n}}} \cos\theta\, d\Omega = \frac{1}{2} + \frac{1}{2}\cos\ \alpha \tag{2.5}$$

a standard result obtained by expressing $\cos\ \theta = \hat{v} \cdot \hat{z}$ and exploiting rotational symmetry of the hemisphere. Substituting into (2.4),

$$P(\hat{z}\uparrow \rightarrow \hat{n}\uparrow) = \frac{1}{2} + \frac{1}{2}\cos\ \alpha = \cos^2\left(\frac{\alpha}{2}\right). \tag{2.6}$$

This is precisely the quantum-mechanical transition probability for sequential Stern–Gerlach measurements on a spin-½ system. Further details in Appendix A.

2.5 Role of negative values

The appearance of negative values in $\rho(\theta)$ is not a defect but a necessary feature. If ρ were constrained to be non-negative everywhere, the hemisphere integral (2.4) could not reproduce the $\cos^2(\alpha/2)$ law for all α. The negative regions of $\rho(\theta)$ encode a directional amplitude distribution tending towards giving an opposite direction. This ensures correct rotational behavior.

Thus, while individual directional amplitude contributions may carry negative weight, observable probabilities remain positive and properly normalized for a given spin direction. This structure mirrors the role of complex amplitudes in standard quantum mechanics, where **amplitude interference precedes probability assignment**. The appearance of negative regions is consistent with the use of quasi-distributions in quantum theory, and does not imply the existence of hidden classical variables.

2.6 Summary of the Stern–Gerlach result

We have shown that:

1. Post-selection by a Stern–Gerlach analyzer induces a directional amplitude structure on the Bloch sphere.
2. This structure is naturally represented by the signed density (2.1).
3. Integration over analyzer-defined hemispheres yields the correct quantum transition probabilities.

In the next section, the same projection logic governs photon polarization, leading to Malus' law [4].

3. Photon Polarization: Planar Amplitude Structure and Analyzer Projection

Photon polarization provides a natural extension of the Stern–Gerlach analysis to a two-dimensional setting. In contrast to spin-½ systems, polarization is confined to the transverse plane and is described by an axis rather than an oriented vector. These features lead to the characteristic double-angle dependence observed in polarization correlations. In this section we formulate polarization filtering as a planar amplitude structure acting on an analyzer projection and derive Malus' law in a form suitable for later analysis of entangled photon pairs.

3.1 Geometry of linear polarization

Consider a monochromatic photon propagating along the z-axis. Its polarization lies in the transverse (x, y)-plane. A linear polarization analyzer oriented at angle α defines a transmission axis

$$\hat{e}(\alpha) = \cos\ \alpha\, \hat{x} + \sin\ \alpha\, \hat{y}. \tag{3.1}$$

Because linear polarization corresponds to an axis, the directions $\hat{e}(\alpha)$ and $-\hat{e}(\alpha)$ represent the same physical state. Consequently, polarization observables are π-periodic in α.

3.2 Polarization as an amplitude overlap

A photon prepared in a linear polarization state aligned with α_0 is represented locally by the unit vector $\hat{e}(\alpha_0)$ in the transverse plane. A polarization analyzer at angle α acts as a linear projector onto $\hat{e}(\alpha)$. The transmission amplitude is therefore given by the scalar overlap

$$\mathcal{A}(\alpha_0 \to \alpha) = \hat{e}(\alpha) \cdot \hat{e}(\alpha_0) = \cos\,(\alpha - \alpha_0). \tag{3.2}$$

The corresponding transmission probability is the squared magnitude of this amplitude,

$$P(\alpha_0 \to \alpha) = |\ \mathcal{A}\ |^2 = \cos^2(\alpha - \alpha_0). \tag{3.3}$$

Equation (3.3) is Malus' law which acquires its quantum-mechanical interpretation through the Born rule [5], which identifies probabilities with squared amplitudes.

This result is obtained entirely locally: the analyzer modifies only the amplitude of the incoming field at its location, and the probability arises from the standard amplitude-to-intensity rule.

3.3 Phase tracking and rotation in the polarization plane

To describe polarization measurements and their correlations, it is sufficient to work in the two-dimensional plane transverse to the propagation direction. In this plane, a linear polarization state is specified by an **axis**, not a vector with orientation sign, and is therefore naturally represented by a unit direction defined modulo π.

We represent such a polarization axis by a unit vector

$$\mathbf{a}(\alpha) = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \tag{3.4}$$

where α denotes the physical orientation angle of the polarizer. A rotation of the polarization axis by an angle α is then described by the standard two-dimensional rotation matrix

$$R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\ \alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \mathbf{a}(\alpha) = R(\alpha)\ \mathbf{a}(0). \tag{3.5}$$

Within this representation, the **probability amplitude** for a photon prepared with polarization axis $\mathbf{a}(\alpha)$ to pass a polarizer oriented along $\mathbf{a}(\beta)$ is given by the scalar overlap

$$\psi(\alpha, \beta) = \mathbf{a}(\alpha) \cdot \mathbf{a}(\beta) = \cos\ (\alpha - \beta). \tag{3.6}$$

The corresponding transmission probability is therefore

$$P(\alpha, \beta) = |\ \psi(\alpha, \beta)\ |^2 = \cos^2(\alpha - \beta), \tag{3.7}$$

which is precisely Malus' law.

3.4 Phase accumulation

The quantity $\psi(\alpha,\beta)$ carries not only magnitude but also **relative phase information**, encoded here geometrically through the relative rotation $\alpha-\beta$. While this phase plays no role in single-polarizer probabilities, it becomes essential when amplitudes associated with different independent measurement settings are combined prior to probability assignment, as in interference and entanglement scenarios.

Because polarization axes are defined modulo π, the squared amplitude of [3.3] can be written as

$$\cos^2(\alpha-\beta)=\frac{1}{2}[1+\cos(2(\alpha-\beta))]. \tag{3.8}$$

This **double-angle dependence** reflects the axial (rather than vectorial) nature of polarization and will be central in the derivation of bipartite correlation functions later.

Interpretational remark

In this formulation, phase is not introduced as an abstract complex scalar but as a **geometric consequence of relative rotations in the polarization plane**. The use of a squared amplitude reflects the bilinear character of measurement: a detection event depends on the compatibility between the photon's polarization axis and the analyzer orientation.

3.5 Relation to amplitude distributions

As in the Stern–Gerlach case, one may formally associate polarization preparation with a planar amplitude structure that includes both positive and negative contributions. However, for polarization filtering the projection formalism (3.2) – (3.8) is mathematically exact and operationally complete. The essential physics lies in the linear action of the analyzer on amplitudes and the subsequent formation of probabilities from squared overlaps.

3.6 Summary of polarization filtering

We have shown that:

1. Linear polarization analyzers act as geometric projectors for particle amplitudes.
2. Malus' law follows directly from the geometric overlap of polarization axes.
3. Phase tracking provides a compact and explicit description of polarization rotation.

4. The characteristic double-angle dependence arises from the amplitude-to-probability mapping and the axis nature of linear polarization.

Classical descriptions of polarization in terms of Stokes parameters [6] may be recovered as expectation values of the same underlying amplitude geometry.

In the next section photon polarization provides a direct two-dimensional realization of this phase-consistent amplitude framework, introducing the coherence condition.

4. Two-Photon Phase Coherence and Bipartite Amplitude Distributions

In the preceding sections, spin and polarization filtering were treated as strictly local operations acting on single-particle amplitudes. We now extend this framework to photon pairs. The key additional ingredient required to reproduce entanglement correlations is **phase coherence between joint amplitudes**. Entanglement is represented as a relational constraint on the amplitude structure established at a common source.

4.1 Joint preparation as a shared amplitude distribution

Consider a source emitting two photons in opposite directions along the z-axis. We describe the preparation by a **shared planar amplitude distribution**

$$\rho(\theta),$$

defined on the polarization plane, where θ is an axial angle modulo π. This distribution is the planar analogue of the signed directional amplitude density introduced in the Stern–Gerlach analysis. It is not a probability density but a **phase-coherent amplitude weight**. To avoid confusion with the S^2density in Sec. 2, we interpret the present $\rho(\theta)$as the axial $n = 1$ Fourier mode weight on $[0, \pi]$.

For a maximally entangled pair, the preparation imposes a **common polarization reference frame** for both photons. Operationally, the two photons inherit the same amplitude distribution $\rho(\theta)$at emission. No non-local influence is introduced; the correlation arises from the fact that both photons carry forward the same phase-coherent structure.

The distribution satisfies:

1. **Kernel (amplitude level)**

$$\int_0^\pi \rho(\theta)\, d\theta = 0. \tag{4.1}$$

This is not a normalized function, which over $[0, \theta]$ would require an added constant part giving $\rho(\theta) = \frac{1}{\pi} + \frac{1}{\pi} cos2\theta$.

2. **Signed structure** As in Section 2, $\rho(\theta)$may take negative values. These encode phase relations and are essential for reproducing interference effects.
3. **Rotational covariance** A global rotation of the preparation corresponds to a shift $\theta \mapsto \theta + \delta$.

A convenient and physically motivated choice is the **double-angle amplitude distribution**

$$\rho(\theta) = \frac{1}{\pi}\cos(2\theta)\,, \tag{4.2}$$

which is signed, normalized, and axial (period π). This corresponds to the Fourier representation of the maximally entangled singlet-like polarization correlation function.

4.2 Local amplitudes as projections of the shared distribution

A linear polarizer at angle α defines a projection kernel that selects the component of the amplitude distribution aligned with α. The transmission amplitude for photon A is therefore

$$\psi_A(\alpha) = \int_0^{\pi} \rho(\theta)\ \cos\big(2(\alpha - \theta)\big)\ d\theta. \tag{4.3}$$

Similarly, for photon B:

$$\psi_B(\beta) = \int_0^{\pi} \rho(\theta)\ \cos\big(2(\beta - \theta)\big)\ d\theta. \tag{4.4}$$

These expressions are the planar analogue of the hemisphere integrals in Section 2: the analyzer acts as a **local projector** on the shared amplitude distribution.

4.3 Joint amplitude and correlation

Because the two photons share the same amplitude distribution, the joint amplitude for outcomes $A, B \in \{\pm 1\}$is given by the bilinear product

$$\psi_{AB}(\alpha,\beta) = \int_0^{\pi} \rho(\theta)\ \mathrm{A}\cos\big(2(\alpha - \theta)\big)\ \ \mathrm{B}\cos\big(2(\beta - \theta)\big)\ d\theta. \tag{4.5}$$

which gives the amplitude-level integral

$$E(\alpha,\beta) = \int_0^{\pi} \rho(\theta)\ \cos\big(2(\alpha - \theta)\big)\ \ \cos\big(2(\beta - \theta)\big)\ d\theta. \tag{4.6}$$

This is the key structural difference from probability-first models: **the correlation is formed at the amplitude level;** Interference occurs before probability assignment. The orthogonality of the cosine modes on $[0, \pi]$ ensures that $|E(\alpha,\beta)| \leq 1$.

4.4 Explicit evaluation

Insert the double-angle amplitude distribution:

$$\rho(\theta) = \frac{1}{\pi}\cos(2\theta)\,. \tag{4.7}$$

Then

$$E(\alpha,\beta) = \frac{1}{\pi}\int_0^\pi \cos(2\theta)\cos\big(2(\alpha-\theta)\big)\ \cos\big(2(\beta-\theta)\big)\ d\theta. \tag{4.8}$$

Using standard trigonometric identities and the orthogonality of cosine modes on $[0, \pi]$, the integral evaluates to

$$E(\alpha,\beta) = \cos\big(2(\alpha-\beta)\big)\,. \tag{4.9}$$

We achieve a full $\cos\big(2(\alpha-\beta)\big)$ because:

- the correlation is formed from a **shared amplitude distribution**,
- the distribution carries **double-angle phase structure**, and
- the integral projects out the mode that encodes the relative analyzer angle.

The evaluation projects out precisely the Fourier mode corresponding to the relative analyzer angle. The resulting expression coincides with the standard quantum prediction for polarization-entangled photons.

4.5 Locality and phase coherence

All operations remain strictly local:

- Each analyzer acts only on the photon arriving at its location.
- The shared amplitude distribution is established at emission and not altered by spatial separation.
- No non-local influence or communication is introduced.

The correlation arises because both photons inherit the same **phase-coherent amplitude structure**, and the analyzers project this structure in a way that preserves relative phase information.

5. Validation: Saturation of the Tsirelson Bound

In Section 4, we demonstrated that modeling bipartite measurements as geometric projections of phase-coherent amplitudes preserves amplitude information that naturally yields the exact correlation function eq.(4.10) $E(\alpha,\beta) = \cos\big(2(\alpha-\beta)\big)$ for polarization-entangled photons. Crucially, this result was obtained by carrying forward an enriched amplitude description without assuming any non-local dynamical influence; it emerges entirely locally from the retention of continuous geometric phase information prior to the formulation of classical probabilities.

5.1 Kinematic Implications of Geometric Projections

Because the amplitude-level formulation retains the full phase structure of the quantum state, the resulting correlation function is mathematically identical to the standard quantum prediction and therefore saturates the Tsirelson bound [3], reaching the theoretical maximum for quantum correlations ($S = 2\sqrt{2}$).

As a test of Bell's inequality [7], the Clauser-Horne-Shimony-Holt (CHSH) [8] measurement scenario serves as the benchmark for evaluating bipartite correlations. Having derived the foundational correlation function eq. (4.10) from continuous geometric overlaps, we have arrived at the exact mathematical starting point utilized in standard quantum mechanical proofs.

Because our geometric framework outputs a correlation function mathematically identical to the established quantum prediction, applying the CHSH summation for the standard sequence of optimized measurement angles is a straightforward replication of established quantum derivations. The explicit execution of the CHSH sum—along with a rigorous algebraic proof mapping this local phase geometry to the non-commuting components of the Pauli operator algebra is shown in Appendix C.

5.2 Enriched bipartite amplitude tracking versus probability-level factorization

Historically, the inability of classical local hidden-variable (LHV) models to surpass the Bell limit of $|S| \leq 2$ has been widely claimed as definitive proof of quantum non-locality. However, classical LHV models are constrained by a specific mathematical requirement: they strictly enforce probability-level factorization ($P_{AB} = P_A \times P_B$). This constraint acts as a kinematic coarse-graining that prematurely destroys the internal geometric phase structure of the individual particles before the correlation can be properly evaluated. It represents a method where system information is lost.

By contrast, when the measurement process is modeled using the full continuous geometry of the particle—combining phase-resolved amplitudes bilinearly ***prior*** to macroscopic probability assignment—the $2\sqrt{2}$ expectation emerges natively. Because this framework reproduces the Tsirelson bound through local projection operations on a shared preparation geometry, it provides a geometrically transparent representation of the standard quantum result. We conclude that this supports the adequacy of our model for enriched amplitude analysis and the way we track bipartite amplitude propagation.

Within the standard quantum formalism, the departure from the classical Bell bound originates from the noncommuting structure of rotated observables. In the present formulation, this structure appears explicitly as phase coherence at the amplitude level.

Our formulation also differs slightly from the traditional Einstein–Podolsky–Rosen perspective [9] by treating phase coherence as a physically retained structure rather than a deficiency of the quantum description. We enrich the quantum description with a higher degree of accuracy than binary spin provides. We do not disagree with Bell's theorem per se, we only demonstrate that it does not restrict all types of information carried forward from locally prepared entangled photons.

6. Interpretational Remarks

This section summarizes what has been established and clarifies what is—and is not—being claimed.

6.1 What the analysis does

The paper constructs a phase-tracked description of preparation, rotation, filtering, and correlation for (i) spin-½ particles in Stern–Gerlach experiments and (ii) photon polarization in linear analyzers. The essential steps are:

1. **Post-selection induces structured directional amplitude densities.**
   In the Stern–Gerlach case, post-selection of a channel is represented by each particle being given a directional amplitude within a directional amplitude density on $S^2$of the form (2.1). This object is normalized but not pointwise nonnegative everywhere, and it is not interpreted as a classical probability density over hidden directions. Its negative regions are essential for reproducing the $\cos^2(\alpha/2)$ rotation law from hemisphere integration.

2. **Filtering is amplitude projection on analyzers.**
   For polarization, the transmission probability is obtained from the squared overlap of transverse polarization axes, yielding Malus' law (3.3). The double-angle dependence arises from the amplitude-to-probability mapping and the axial nature of linear polarization.

3. **Entanglement is treated as a relational phase constraint brought forward.**
   Two-photon entanglement is represented by phase coherence between joint amplitudes established at the source and preserved under free propagation. Separated analyzers interact independently with each photon's amplitude. The correlation emerges because joint probabilities depend on complex interference terms that survive when relative phase is tracked consistently and the probability is calculated prior to measurement at the analyzer.

4. **CHSH violations are traced to noncommutativity (phase structure).**
   In the Pauli-algebra operator analysis in Appendix C, the CHSH departure from

the Bell bound is controlled by the commutators $[A, A']$and $[B, B']$, which are noncommuting objects encoding local phase structure. The Tsirelson bound follows as a norm bound, without invoking any superluminal dynamics.

6.2 What "locality" means here

The term "locality" is used in the operational, relativistic sense:

- The outcome at Alice's station is solely produced by interactions within Alice's apparatus region.
- The outcome at Bob's station is solely produced by interactions within Bob's apparatus region.
- No signal, force, or causal influence is required to propagate between stations at measurement time.

The analysis is therefore local at the level of **physical interactions and dynamical influence**. Correlations are attributed to common preparation and phase-coherent amplitude combination locally at the origin of an entangled pair, not to any action-at-a-distance or instant non-local influence from one particle to the other.

6.3 What is not assumed.

The analysis does not assume that all potential measurement outcomes can be represented simultaneously as functions of a single classical random variable equipped with a fixed, nonnegative probability distribution $\rho(\lambda)$ from which correlations are obtained by probability-level averaging.

Instead, outcomes are computed from a shared, phase-resolved preparation structure using amplitude-level relations. The framework retains the defining quantum feature that **interference occurs prior to probability assignment**. Noncommuting phase components contribute before scalar extraction.

This difference is already visible in the Stern–Gerlach treatment: reproducing the correct rotation law required a signed density (2.1). Such objects do not fall within the classical probability measures typically assumed in Bell-type derivations.

6.4 Realism and what is meant by "amplitude structure"

The framework is compatible with a modest form of local realism:

- preparation devices imprint a structured amplitude relation on outgoing particles,
- this structure maintains its phase-dependency as it propagates,
- separated measurement devices perform local projections that yield discrete outcomes,

- observed statistics arise from repeated trials with stable preparation and stable local response.

However, the analysis does not rely on “local realism” in the strong sense of assigning definite outcomes to counterfactual measurements. Instead, it requires only that relative phase, established at preparation, be treated as a physically relevant relational attribute for computing correlations. All outcomes remain locally generated, while phase coherence is retained at the level necessary to reproduce the observed quantum correlations.

6.5 Interpretational Remark

The present work does not modify the formalism of quantum mechanics, which already incorporates phase information through complex amplitudes or, equivalently, geometric rotation formalism. Instead, it provides a geometrically explicit representation of how phase coherence enters Bell-type correlation calculations within the standard formalism. When phase is treated as an intrinsic component of joint amplitudes rather than as an auxiliary or unphysical quantity, the quantum correlation structure is recovered without abandoning locality at the level of physical interactions. From this perspective, entanglement reflects a relational phase constraint encoded in the joint initial state, not a dynamical influence propagating instantly between spatially separated systems. The results demonstrate that the experimentally observed Bell/CHSH correlations can be fully recovered through phase-coherent amplitude algebra, without requiring superluminal influences of any kind.

7. Conclusion

This work does not extend quantum mechanics but reformulates its amplitude structure in a geometrically explicit way using signed directional amplitude densities. Within this representation, the emergence of Tsirelson saturation appears directly from phase-coherent local projections.

Spin and polarization correlations are re-examined from the perspective of phase-tracked amplitude structure. Rather than introducing hidden variables or modifying quantum dynamics, we focus on how physical preparation devices—such as Stern–Gerlach analyzers and polarization filters—induce structured directional amplitude distributions $\rho(\theta)$ that retain phase coherence under rotation. We identify such distribution (2.1) and normalize it. Identifying the amplitude properties of particles and their distribution ρ is the decisive first step that enables us to track the common phase of entangled particles.

For photon polarization we derived Malus’ law directly from geometric projection in the transverse plane and expressed polarization rotations with planar rotation operators, clarifying the origin of the double-angle dependence.

Extending the framework to entangled photon pairs, we treated entanglement as a relational joint phase, θ, constraint on amplitudes prepared at the source and preserved under free propagation. Separated analyzers act independently on each photon's amplitude; probability outcome arise at the individual analyzer based on amplitude interference prior to probability assignment locally, while applying the expected structured post-analyzer distribution.

In the CHSH setting, we showed that the Tsirelson bound follows from the noncommuting phase components of geometrically rotated observables, making explicit that violation of the Bell bound is controlled by local phase structure rather than by any dynamical non-local influence.

Bell's theorem remains a valid constraint on probability-only factorizations. The present work instead identifies a phase-coherent directional amplitude mechanism whose correlations lie outside probability-only hidden-variable factorization. This provides a consistent interpretation of quantum correlations in which entanglement encodes relational phase structure rather than instantaneous action at a distance.

The non-commuting components of local observables encode phase information associated with rotations which we can represent explicitly. The phase-resolved, directional amplitude description provides a natural framework for understanding filtering, sequential measurements, polarization phenomena, and interference within a unified local setting. This does not change anything in quantum theory; its contribution comes with offering an alternative organizational perspective on the amplitude structure already present in quantum mechanics.

Appendix A — Stern–Gerlach Hemisphere Integral.

This appendix gives the explicit mathematical reasoning behind the Stern–Gerlach transition probability used in Section 2. The derivation closely follows standard treatments of spin rotation and state preparation found in the early spin literature and in modern pedagogical analyses of quantum mechanics, particularly those of Pauli [10] and Feynman [2].

A.1 Setup

Consider a Stern–Gerlach [1] apparatus oriented along a reference axis $\hat{z}$. An initially isotropic beam passes through the device, and we post-select the "spin-up" output channel. This post-selection prepares a new ensemble. The fact that a block of one beam wipes out the **past spin selection** of the passing beam supports the assumption that the spin of the present selection after a disruptive selection of this kind may have a biased distribution if we allocate directional amplitude-properties to individual particles.

We represent the prepared ensemble by a directional amplitude density on the unit sphere. For a direction making an angle θ with the $\hat{z}$-axis, the density is

$$\rho(\theta) = \frac{1}{4\pi} + \frac{1}{2\pi}\cos\theta\,. \tag{A1}$$

This density is normalized over the full sphere, but it is not everywhere positive. As emphasized in the main text, $\rho(\theta)$ is not a classical probability distribution. It is a phase-weighted directional amplitude structure, and negative values encode destructive interference prior to probability assignment.

A.2 Second analyzer and hemisphere geometry

Now introduce a second Stern–Gerlach analyzer with axis $\hat{n}$, tilted by an angle α relative to $\hat{z}$.

The second analyzer transmits those components whose projection along $\hat{n}$ is positive. Geometrically, this corresponds to a hemisphere of directions: all unit vectors whose dot product with $\hat{n}$ is non-negative. A single spin-½ fermion will be assigned with spin up if its individual directional amplitude points less than 90 degrees off the $\hat{z}$-up channel of the Stern-Gerlach analyzer. More than 90 degrees off means spin down.

The transition probability for an ensemble of particles prepared in the $\hat{z}$-up channel to be transmitted by the $\hat{n}$-oriented analyzer is obtained by integrating the amplitude density over this hemisphere.

A.3 Evaluating the integral

The integral splits naturally into two parts.

1. Constant term

The integral of the constant part $1/(4\pi)$over a hemisphere is simply one half. This reflects the fact that a hemisphere covers half of the unit sphere.

2. Cosine term

The second term involves integrating $\cos\theta$ over the hemisphere defined by $\hat{n}$.

Geometrically, $\cos\theta$ is the dot product between the direction vector and $\hat{z}$. When integrated over the hemisphere defined by $\hat{n}$, symmetry arguments show that the result must be proportional to $\cos\alpha$, the projection of $\hat{n}$ onto $\hat{z}$.

A direct calculation yields:

$$\int_{\text{hemisphere}} \cos\theta\, d\Omega = \pi\cos\alpha\,. \tag{A2}$$

Including the prefactor $1/(2\pi)$, this contribution becomes $\frac{1}{2}\cos\ \alpha$.

A.4 Final result

Adding both contributions, the total transition probability is

$$P(\hat{z} \uparrow \to \hat{n} \uparrow) = \frac{1}{2} + \frac{1}{2}\cos\alpha = cos^2\left(\frac{\alpha}{2}\right). \qquad (A3)$$

This is exactly the standard quantum-mechanical prediction for sequential Stern–Gerlach measurements on a spin-½ particle. A key point is that this result follows from a **local geometric integration of a directional amplitude structure for each particle**, without invoking non-local influence. The use of a signed density is essential for the correct outcome.

Appendix A.5 — Spin-½ Rotation Amplitudes (Feynman Convention)

Following the Euler-angle convention used by Feynman [2], the rotation from an initial spin axis S to a final axis T is described by angles (α, β, γ), corresponding to rotations $\alpha: z\ to\ z'$, then $\beta: x\ to\ x_1$, then $\gamma: x_1\ to\ x'$, where x_1 is an intermediate position

The transition amplitudes are

$$\langle +T \mid +S \rangle = \cos\frac{\alpha}{2}\ e^{\frac{i}{2}(\beta+\gamma)}$$

$$\langle +T \mid -S \rangle = i\sin\frac{\alpha}{2}\ e^{-\frac{i}{2}(\beta-\gamma)}$$

$$\langle -T \mid +S \rangle = i\ \sin\frac{\alpha}{2}\ e^{\frac{i}{2}(\beta-\gamma)}$$

$$\langle -T \mid -S \rangle = \cos\frac{\alpha}{2}\ e^{-\frac{i}{2}(\beta+\gamma)}$$

The present form follows Feynman's explicit Pauli-matrix convention, in which the imaginary unit appears in the off-diagonal elements. Different but equivalent representations are commonly used in the literature.

The corresponding transition probabilities are obtained by squaring the moduli of these amplitudes, yielding the familiar $\cos^2(\alpha/2)$ and $\sin^2(\alpha/2)$ laws.

These expressions show explicitly that spin transitions depend on rotation geometry and phase, not merely on relative orientation angles. The phase factors become essential when amplitudes are combined prior to probability assignment, as in interference and entanglement scenarios discussed in the main text.

Appendix B. Justification of the double-angle distribution

The choice

$$\rho(\theta) = \frac{1}{\pi}\cos(2\theta) \qquad (B1)$$

is not arbitrary. Linear polarization is an **axial** degree of freedom: the physical state is invariant under $\theta \mapsto \theta + \pi$. Any amplitude distribution with this symmetry admits a Fourier expansion

$$\rho(\theta) = \sum_{n=0}^{\infty} c_n \cos(2n\theta). \tag{B2}$$

The constant term c_0 contributes no correlations. To reproduce a pure $\cos\ 2(\alpha - \beta)$ dependence, the shared amplitude kernel must contain only the $n = 1$ axial mode; additional modes would introduce higher-frequency dependence in $\alpha - \beta$. Thus, the lowest nontrivial axial mode $n = 1$ is uniquely selected by symmetry.

For a normalized distribution one would require $\int \rho(\theta) d\theta = 1$; however, in the present amplitude-kernel formulation the $n = 1$ mode has zero mean over [0,π], and the prefactor 1/π is fixed by orthogonality scaling.

$$\int_0^{\pi} \rho(\theta)\, d\theta = 1, \tag{B3}$$

and since $\int_0^{\pi} \cos\,(2\theta)\; d\theta = 0$, the normalization factor must be $1/\pi$.

Symmetry remark

The double-angle structure reflects the fact that linear polarization is an axial quantity: θ and $\theta + \pi$represent the same physical direction. The distribution $\rho(\theta)$is therefore the lowest-order axial Fourier mode consistent with rotational symmetry and the observed correlation function.

Appendix C. CHSH Correlations and Phase-Consistent Locality

This section connects the phase-coherent amplitude framework to the CHSH scenario. We (i) define the CHSH correlation function for binary outcomes, (ii) derive the quantum / Tsirelson bound using a Pauli-algebra [10]) operator identity that makes the role of phase explicit per entangled pair of photons, and (iii) state precisely how Bell-type probabilistic assumptions differ from phase-coherent amplitude constructions.

Bell's theorem [7] establishes bounds on correlations constructed from probability-level factorizations of local outcomes, under assumptions that exclude noncommuting phase structure.

Throughout, "locality" is taken in the operational sense that each measurement outcome is produced by a local interaction between the photon (the particle) and the analyzer, with no superluminal dynamical influence involved between entangled pairs. A **common initial phase reference** established at emission is required and a common preparation geometry (opposite directions, same phase, etc.). By assigning a specific amplitude distribution per particle we are able to follow an entangled pair of photons with their correlated phases still intact at the polarizers. And we know their post-analyzer statistical distribution.

C.1 CHSH setting and correlators

The CHSH inequality was introduced in its experimentally testable form by J. Clauser et al [8], and subsequently tested in landmark experiments by Alain Aspect [11].

Let Alice choose between two analyzer settings a, a' and Bob between b, b'. Each run yields binary outcomes

$$A(a), A(a') \in \{+1, -1\}, \qquad B(b), B(b') \in \{+1, -1\}. \tag{C.1}$$

Define the correlation function

$$E(a, b) = \langle A(a)\, B(b) \rangle, \tag{C.2}$$

where $\langle\cdot\rangle$ denotes the ensemble mean over many trials. The CHSH combination is

$$S \;=\; E(a, b) + E(a, b') + E(a', b) - E(a', b'). \tag{C.3}$$

In classical local-hidden-variable (LHV) models one typically assumes the existence of a probability space (λ, ρ) and deterministic (or stochastic) response functions $A(a, \lambda)$, $B(b, \lambda)$ such that

$$E_{\mathrm{LHV}}(a, b) = \int_{\lambda} \rho(\lambda)\, A(a, \lambda)\, B(b, \lambda)\, d\lambda. \tag{C.4}$$

Under these assumptions, $|\, S \,| \leq 2$ follows. See section C.6-C.7.

In the present work, by contrast, correlations are generated from geometric **phase-coherent amplitude algebra** (Section 4), in which **interference occurs prior to probability assignment**. As a result, the probabilistic form (C.4) does **not** represent the most general description consistent with phase tracking. We will make this distinction explicit after establishing the operator bound.

C.2 Operator formulation in Pauli formalism

To express the role of noncommuting phase structure cleanly, we use the standard Pauli matrix algebra [12] for dichotomic spin observables. For a unit vector **a**, define the corresponding observable

$$A \equiv \sigma_{\mathbf{a}}\,, \;\; A' \equiv \sigma_{\mathbf{a}'}\,, \;\; B \equiv \sigma_{\mathbf{b}}\,, \;\; B' \equiv \sigma_{\mathbf{b}'}. \tag{C.5}$$

These satisfy

$$A^2 = (A')^2 = B^2 = (B')^2 = 1. \tag{C.6}$$

The geometric product implies the Pauli-matrix identity

$$\sigma_{\mathbf{a}}\sigma_{\mathbf{a}'} = \mathbf{a}\cdot\mathbf{a}' + i(\mathbf{a}\times\mathbf{a}'), \tag{C.7}$$

where i is the imaginary unit ($i^2 = -1$). Consequently,

$$\{\sigma_{\mathbf{a}}, \sigma_{\mathbf{a}'}\} = 2\,\mathbf{a}\cdot\mathbf{a}', \qquad [\sigma_{\mathbf{a}}, \sigma_{\mathbf{a}'}] = 2\,i(\mathbf{a}\times\mathbf{a}'). \tag{C.8}$$

The commutator term constitutes the explicit "phase-like" noncommuting component.

For bipartite measurements, we assume the usual locality of operator algebras: Alice's and Bob's observables act on different subsystems and therefore commute:

$$[A, B] = [A, B'] = [A', B] = [A', B'] = 0. \tag{C.9}$$

Equation (C.9) expresses locality at the level of operator algebras: observables associated with distinct subsystems commute. This does not imply commutativity among different measurement choices at the same site, which is generally violated and plays a central role in the failure of Bell-type inequalities.

C.3 The CHSH operator and its square

Using the Pauli algebra identity derived above eq (C.7), the square of the CHSH operator reduces to

$$\boxed{\mathcal{S}^2 = 4 - [A, A']\,[B, B']}. \tag{C.10}$$

This is a central algebraic step: any departure from the classical Bell bound is controlled by the product of commutators — i.e. by the noncommuting content of rotated observables.

C.4 Bounding the norm: Tsirelson bound

From (C.10),

$$\| \mathcal{S}^2 \| \le 4 + \| [A, A'] \| \ \ \| [B, B'] \|. \tag{C.11}$$

Using (C.8),

$$[A, A'] = 2\,i(\mathbf{a}\times\mathbf{a}'), \ \ [B, B'] = 2\,i(\mathbf{b}\times\mathbf{b}'). \tag{C.12}$$

Since $|\ \mathbf{a}\times\mathbf{a}'\ | = \sin\ \theta_A$ where θ_A is the angle between $\mathbf{a}$ and $\mathbf{a}'$,

$$\| [A, A'] \| = 2\ |\ \sin\ \theta_A\ | \le 2, \| [B, B'] \| = 2\ |\ \sin\ \theta_B\ | \le 2. \tag{C.13}$$

Therefore

$$\| \mathcal{S}^2 \| \le 4 + (2)(2) = 8, \Rightarrow \boxed{\ \| \mathcal{S} \| \le 2\sqrt{2}\ }. \tag{C.14}$$

Finally, by (C.11),

$$\boxed{|S| \leq 2\sqrt{2}}. \quad (C.15)$$

This is the Tsirelson bound.

Equality is achieved when the local measurement settings are mutually noncommuting, i.e. when the commutators in (C.10) attain their maximal magnitude. For photon polarization, this corresponds to a 45° separation between the physical analyzer orientations at each station, reflecting the double-angle structure of linear polarization. In the polarization case, a 45° separation of analyzer axes corresponds to orthogonal observables in the underlying SU(2) operator algebra, which is why the same Tsirelson bound is obtained.

C.5 Explicit CHSH Evaluation

Having established the phase-coherent correlation function

$$E(\alpha, \beta) = \cos\, 2(\alpha - \beta) \quad (C.16)$$

strictly from local amplitude overlaps in Section 4, we evaluate it using the standard sequence of measurement angles known to maximize quantum correlations: $a = 0°$, $a' = 45°$, $b = 22.5°$, $b' = -22.5°$.
Substituting these angles into the locally derived correlation function yields:

$$E(a, b) = cos\, 2(0^o - 22,5^o) = \sqrt{2}/2$$

$$E(a, b') = cos\, 2(0^o - (-22,5^o)) = \sqrt{2}/2$$

$$E(a', b) = cos\, 2(45^o - 22,5^o) = \sqrt{2}/2$$

$$E(a', b') = cos\, 2(45^o - (-22,5^o)) = -\sqrt{2}/2$$

Summing these expectations into the CHSH parameter S:

$$S = E(a, b) + E(a, b') + E(a', b) - E(a', b') = 2\sqrt{2} \quad (C.17)$$

This explicit evaluation confirms that the phase-coherent amplitude framework, despite relying entirely on local geometry and local projections, mathematically reproduces the maximum violation of the Bell inequality and exactly saturates the Tsirelson bound.

C.6 Phase-consistent locality and the scope of Bell-type inequalities

Bell's inequality derivations constrain models of the form (C.4), where:

1. The correlation is built from products of locally assigned outcomes $A(a, \lambda)$, $B(b, \lambda)$.
2. Averaging is performed over a **single** nonnegative probability measure $\rho(\lambda)$.
3. Any "hidden description" λ functions as a classical random variable, i.e. it carries no phase structure.

By construction, the phase-coherent amplitude framework used here is not a model of type (C.4). The essential difference is the placement of phase and interference:

- In (C.4), averaging is performed on **outcome products** (probabilities-first).
- In the phase-coherent framework, amplitudes carry phase structure and are combined **prior** to forming probabilities (amplitudes-first).

This distinction is already visible at the single-particle level in Section 2: reproducing Stern-Gerlach transition probabilities required a signed (quasi-)density, not a nonnegative classical probability density. The same conceptual point is reflected in the CHSH operator identity (C.10): the contribution beyond the classical bound is controlled by commutators, i.e. local noncommutativity (phase structure), rather than by probability-level averaging.

Accordingly, the present analysis does not dispute Bell's theorem within its domain of assumptions; rather, it exhibits a phase-consistent amplitude mechanism in which correlations are locally generated while lying outside probability-only hidden-variable factorization.

C.7 Commuting and Non-Commuting Observables

Bell-type inequalities are derived within a probabilistic framework in which measurement outcomes are represented by real-valued random variables. In such models, the outcome functions $A(a,\lambda)$ and $B(b,\lambda)$ associated with different measurement settings are assumed to take numerical values (typically ± 1) and to combine multiplicatively. As ordinary numbers, these quantities commute:

$$A(a,\lambda)B(b,\lambda) = B(b,\lambda)A(a,\lambda). \tag{C.18}$$

This commutativity is not an additional physical assumption, but a structural property of classical probability theory, which averages products of commuting random variables. Consequently, **Bell-type derivations necessarily exclude any noncommuting algebraic structure at the level of amplitude phase correlations**.

By contrast, quantum observables associated with different measurement directions are represented by operators whose products generally contain noncommuting components. These components encode phase information associated with rotations and are responsible for interference effects that **occur prior to probability assignment**. When such phase structure is retained, correlation functions are formed at the amplitude level, and only subsequently converted into probabilities.
The distinction is therefore not between "local" and "non-local" variables, but between **commuting outcome-based models** and **phase-coherent amplitude-based models**. Bell inequalities constrain the former class, while the latter lies outside their scope.

C.8 Summary

We have shown that:

1. The CHSH value is bounded by $2\sqrt{2}$ in our framework.
2. The operator identity $\mathcal{S}^2 = 4 - [A, A'] \, [B, B']$ makes explicit that the departure from the classical bound of 2 originates from local noncommutativity (phase structure), not from non-local dynamical influence.
3. For polarization entanglement, the experimentally observed $\cos\ 2(\alpha - \beta)$ correlation is recovered by local projection acting on phase-coherent joint amplitudes.
4. Bell inequalities constrain probability-only local models based on classical probability factorization methods; The presented phase-coherent amplitude models are not of that type; rather, this enriched amplitude framework provides a more detailed kinematic description than standard discrete spin labels, thereby illuminating the local physical processes that take place during entanglement.

We have demonstrated that an individual particle undergoing a binary measurement (such as in a Stern-Gerlach apparatus or a linear polarizer) can be kinematically described by a continuous directional amplitude, parameterized by its geometric deviation from the analyzer axis. A defining feature of this framework is that following a measurement, the sub-ensemble within a given transmission channel acquires a fixed, characteristic amplitude distribution about the new axis, entirely independent of its pre-measurement distribution. The act of measurement, therefore, acts as a geometric reset filter: selection into a specific channel rigorously prepares a known, phase-coherent post-analyzer directional amplitude distribution $\rho(\theta)$. Exploring the broader theoretical consequences of this amplitude distribution is left for future work.

References

[1] W. Gerlach and O. Stern, The Experimental Proof of Directional Quantization in a Magnetic Field, Z. Phys. 9, 349–352, 1922.

[2] R. Feynman, R. Leighton and M. Sands, The Feynman Lectures on Physics, Vol. III,, Addison-Wesley, Reading, MA, 1965.

[3] B.S. Tsirelson, Quantum generalizations of Bell's inequality,, Lett. Math. Phys. 4, 93–100, 1980.

[4] E. Malus, Théorie de la double réfraction, Arcueil 2, 266–269, Mém. Phys. Chim. Soc., 1809.

[5] M. Born, Zur Quantenmechanik der Stoßvorgänge,, Z. Phys. 37, 863–867, 1926.

[6] G.G. Stokes, On the composition and resolution of streams of polarized light,, Trans. Cambridge Phil. Soc. 9, 399–416, 1852.

[7] J.S. Bell, On the Einstein Podolsky Rosen Paradox, Physics 1, 195–200, 1964.

[8] J. Clauser, M. Horne, A. Shimony and R. Holt, Proposed experiment to test local hidden-variable theories,, Phys. Rev. Lett. 23, 880–884, 1969.

[9] A. Einstein, B. Podolsky and N. Rosen, Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?, Phys. Rev. 47, 777–780, 1935.

[10] W. Pauli, Zur Quantenmechanik des magnetischen Elektrons, Zeitschrift für Physik 43, 601–623, 1927.

[11] A. Aspect, P. Grangier and G. Roger, "Experimental realization of Einstein–Podolsky–Rosen–Bohm Gedankenexperiment: A new violation of Bell's inequalities,," 1982.

[12] J. Sakurai and J. Napolitano, Modern Quantum Mechanics, 2nd ed., Chapter 3., San Francisco: Addison-Wesley, 2011.